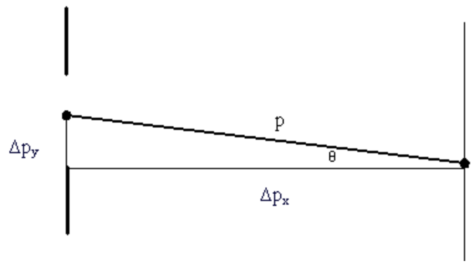
Heisenberg's Uncertainty Principle

When particles are sent from a source towards a single slit, they have momentum in the x direction only (i.e. straight ahead). If they are deflected, then they acquire momentum in the y direction as well (i.e. to either side). Considering the bright region in the centre of the diffraction pattern only, where about 85% of the photons land, a mathematical relationship can be determined that will take into account the uncertainty of where the photons will land.

Consider a particle passing through a single slit with a given momentum.



We can break the total momentum of the particle into its x and y components giving

$$\frac{\Delta p_y}{\Delta p_x} = \tan\theta$$

From the single slit experiment we know that

$$\sin\theta = \frac{\lambda}{w}$$

and the de Broglie wavelength is given by

$$\lambda = \frac{h}{p}$$

Therefore, if θ is very small tan θ ~ sin θ and equation (1) becomes:

$$\frac{\Delta p_y}{\Delta p_x} \approx \frac{h}{w\Delta p_x}$$

But w is just Δy, therefore

$$\Delta p_y \Delta y \approx \hbar$$

With further analysis, Heisenberg concluded that

$$\Delta p_y \Delta y \geq \hbar \qquad (14)$$

where $\hbar = \dfrac{h}{2\pi}$

Δp is the momentum of the particle

Δy is the position of the particle

By relating momentum to energy, the equation becomes

$$\Delta E \Delta t \geq \hbar \qquad (15)$$

where ΔE is the energy of the particle

Δt is the length of time the particle has that energy

Therefore, if we know the position of a particle, we cannot know its momentum and vice versa. Similarly, if we know the energy of the particle, we cannot know the length of time it has that energy and vice versa.

Examples:

1. In a diffraction pattern, an electron is deflected with a speed of 1000 m/s in the y direction. How precisely do we know the position in the slit?

2. An alpha particle is emitted from the decay of Uranium 238. If this particle has energy of 34keV, what is the uncertainty in its position?